Mark Scheme 4726
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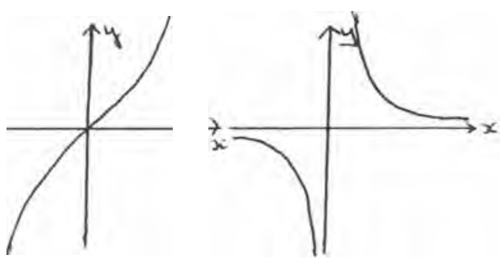
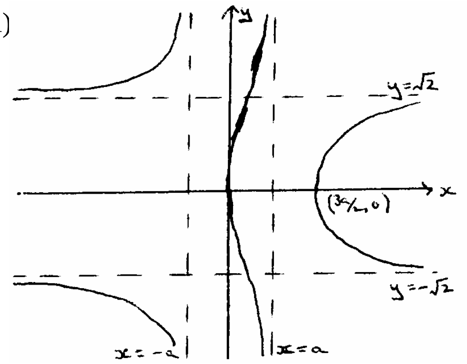
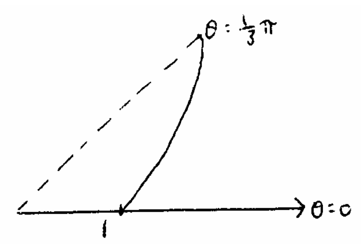
1 (i) f(O) = In 3 f	Bl
f'(0) = $^1/_3$	Bl
f"(O) = -'/$_9$ *A.G.*	B1 Clearly derived
(ii) Reasonable attempt at Maclaurin	Ml Form In3 + ax + bx^2, with a,b related to f " f'
f(x) = In 3 + $^1/_3x$-$^1/_{18}x^2$	*Al* √*J* On their values off' and f"
	SR Use *In(3+x)* = In3 + In(1 +$^1/_3$ *x)* Ml Use Formulae Book to get
	In3 + *Y3X - Y2(VJX)2* =
	In3 + *Y3X - 1/lgX2* Al
2 (i) f(0.8) = - 0.03, f(0.9) = +0.077 (accurately e.g. accept -0.02 t0 -0.04)	B1
Explain (change of sign, graph etc.)	B1
	SR Use x = √*J(tan^{-l}x)* and compare x to √J(tan^{-l} x) for x=*0.8*, 0.9 B 1
	Explain "change in sign" B 1
(ii) Differentiate two terms	B1 Get *2x* - I *I(* 1 +x^2*)*
Use correct form of Newton-Ra ph son with 0.8, using their f *'(x)*	Ml 0.8 - *f(0.8)/f '(0.8)*
Use their N-R to give one more approximation to 3 d.p. minimum	Ml√
Get x = 0.835	Al 3d.p. - accept answer which rounds
3 (i) Show area of rect. = $^1/_4$ (e$^{l/16}$ + e$^{1/4}$ + e$^{9/16}$ + e^{l})	Ml Or numeric equivalent
Show area = 1.7054	Al At least 3 d.p. correct
Explain the < 1.71 in terms of areas	Bl AG. Inequality required
(ii) Identify areas for > sign	B1 Inequality or diagram required
Show area of rect. = $^1/_4$ *(e^o* + *e^{ll16}* + e$^{1/4}$ + e$^{9/16}$)	Ml Or numeric evidence
Get A > 1.27	Al cao; or answer which rounds down
4 (i) 	BI Correct shape for sinh x B1 Correct shape for cosech x B1 Obvious point *(dy/dx* ≠O)/asymptotes clear
(ii) Correct definition of sinh x	B1 May be implied
Invert and mult. by *eX* to AG.	B1 Must be clear; allow *2/(eX* - e *-X)* as mimimum simplification
Sub. u = e^x and du = e^x dx	M1 Or equivalent, all x eliminated and not dx = du
Replace to *2/(u^2* - 1) *du*	Al
Integrate to *aln((u - l)/(u* + 1)	A1√ Use formulae book, PT, or atanh^{-l}u
Replace u	Al No need for c

5 (i) Reasonable attempt at parts Get	M1 Involving second integral Al
xnsin x - $\int$ sin x. nx^{n-1} dx	M1
Attempt parts again Accurately	Al
Clearly derive AG.	A1 Indicate $(^1/_2\pi)^n$ and 0 from limits
(ii) Get $I_4 = (^1/_2\pi)^4 - 12I_2$ or $I_2 = (^1/_2\pi)^2 - 2I_0$	B1
Show clearly $I_0 = 1$	B1 May use I_2
Replace their values in relation Get	M1
$I_4 = {}^1/_{16}\pi^4 - 3\pi^2 + 24$	A1 cao
6 (i) $x = \pm a$, $y = 2$	B1, B1, B1 Must be =; no working needed
(ii) 	B1 Two correct labelled asymptotes ∥ Ox and approaches B1 Two correct labelled asymptotes ∥ Oy and approaches B1 Crosses at $(^3/_2a,0)$ (and (0,0) - may be implied B1 90° where it crosses Ox; smoothly B1 Symmetry in Ox
7 (i) Write as $A/t + B/t^2 + (Ct + D)/(t^2 + 1)$	M1 Allow $(At+B)/t^2$; justify $B/t^2 + D/(l + t^2)$ if only used
Equate $At(t^2+1) + B(t^2+1) + (Ct+D)t^2$ to $1 - t^2$	M1√
Insert t values / equate coeff.	M1 Lead to at least two constant values
Get A = C = 0, B = L D =-2	Al
	SR Other methods leading to correct PF can earn 4 marks; 2 M marks for reasonable method going wrong
(ii) Derive or quote cos x in terms of t	Bl
Derive or quote dx = 2 $dt/(1 + t^2)$	B1
Sub. in to correct P.F.	M1 Allow $k\,(l-t^2)/((t^2(l+t^2))$ or equivalent
Integrate to $-1/t$ -2 $\tan^{-1}t$	Al√ From their k
Use limits to clearly get AG.	Al
8 (i) Get $(e^y - e^{-y})/(e^y + e^{-y})$	B1 Allow $(e^{2Y}-1)/(e^{2y}+1)$ or if x used
(ii) Attempt quad. in e^Y	M1 Multiply by e^Y and tidy
Solve for e^Y	M1
Clearly get AG.	Al
(iii) Rewrite as tanh $x = k$	M1 SR Use hyp def^n to get quad. in e^X M I
Use (ii) for x = $^1/_2$ 1n 7 or equivalent	Al Solve $e^{2x} = 7$ for x to $^1/_2$ 1n 7 Al
(iv) Use of log laws	Bl One used correctly
Correctly equate 1n A = 1n B to $A = B$	M1 Or $1n(^A/_B) = 0$
Get $x = \pm\, ^3/_5$	Al

9 (i)		B1 Shape for correct θ; ignore other θ Used; start at $(r,0)$ B1 θ=0, r=1 and increasing r
(ii)	U se correct formula with correct r	B1
	$\int \sec^2 x \, dx = \tan x$ used	B1
	Quote $\int 2 \sec x \tan x \, dx = 2\sec x$	B1 Or sub. correctly
	Replace $\tan^2 x$ by $\sec^2 x - 1$ to integrate	M1
	Reasonable attempt to integrate 3 terms And to use limits correctly	M1
	Get $\sqrt{3} + 1 - \frac{1}{6}\pi$	Al Exact only
(iii)	Use $x = r\cos\theta$, $y = r\sin\theta$, $r = (x^2+y^2)^{1/2}$	M1
	Reasonable attempt to eliminate r,θ	M1
	Get $y = (x-1)\sqrt{(x^2+y^2)}$	A1 Or equivalent